

Scalar Casimir effect in a high-dimensional cosmic dispiration spacetime

H. F. Mota*, E. R. Bezerra de Mello† and K. Bakke‡

*Departamento de Física, Universidade Federal da Paraíba
58059-900, Caixa Postal 5008, João Pessoa, PB, Brazil*

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Abstract

In this paper we present a complete and detailed analysis of the calculation of both the Wightman function and the vacuum expectation value of the energy-momentum tensor that arise from quantum vacuum fluctuations of massive and massless scalar fields in the cosmic dispiration spacetime, which is formed by the combination of two topological defects: a cosmic string and a screw dislocation. This spacetime is obtained in the framework of the Einstein-Cartan theory of gravity and is considered to be a chiral space-like cosmic string. For completeness we perform the calculation in a high-dimensional spacetime, with flat extra dimensions. We found closed expressions for the the energy-momentum tensor and, in particular, in (3+1)-dimensions, we compare our results with existing previous ones in the literature for the massless scalar field case.

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1 Introduction

Casimir effect is a macroscopic quantum manifestation of vacuum fluctuations by virtue of the imposition of boundary conditions on relativistic fields, namely, scalar, spinor and electromagnetic fields [1–3]. This effect was first predicted in 1948 by Casimir [4] who investigated the quantum vacuum fluctuations of the electromagnetic field occurring inside two large parallel conducting plates. His result was first confirmed by Sparnaay in 1958 [5] and later by other researchers [6] (see also [7–12] for experimental results involving curved-plane surface configurations). It is well known that the modification on the quantum vacuum fluctuations of fields are caused not only by the imposition of boundary conditions but also by the nontrivial topology of the spacetime [1, 2]. Moreover, the current status of the Casimir effect establishes its reality and motivates the study of quantum vacuum fluctuations of other observables rather than only the Casimir energy density.

In this direction, two types of spacetimes with nontrivial topology have mainly been considered in the literature, that is, screw dislocation (which is associated with torsion) and cosmic

*E-mail: hmota@fisica.ufpb.br

†E-mail: emello@fisica.ufpb.br

‡E-mail: kbakke@fisica.ufpb.br

string. The latter is a linear topological defect supposed to be formed in the early Universe after it went through phase transitions and is predicted in the context of extensions of the Standard Model of particle physics [13,14] as well as in the context of string theory [15,16]. Once formed, cosmic strings can evolve in the Universe and contribute to a variety of astrophysical, cosmological and gravitational phenomena [15–17]. From the static point of view, the presence of a idealized thin, straight and very long cosmic string produces a spacetime with a conical topology with a planar angle deficit on the two-surface orthogonal to it given by $\Delta\varphi = 8\pi G\mu_0$, where G is the Newton’s gravitational constant and μ_0 the cosmic string linear energy density [13,14].

The investigation of quantum vacuum fluctuations of physical observables, such as the energy-momentum tensor, due to the conical topology of the cosmic string spacetime can be found in Refs. [18–28]. Another physical observable of great interest is the induced quadri-current by quantum vacuum fluctuations associated to charged matter fields and which has been investigated in Refs. [29–33]. In the context of a linearized model of lightcone fluctuations [34], a delay or advance in time in the propagation of photons arises as a consequence of quantum vacuum fluctuations due to the conical topology of the cosmic string spacetime. The analysis of this observable, i.e. the shift in time, was considered in Ref. [35].

Concerning the screw dislocation, it is also considered to be a line-like topological defect found in the context of theories of solid and crystal continuum media and seen as a type of Volterra distortion [36]. In the framework of the Einstein-Cartan theory of gravity, on the other hand, the merger of a screw dislocation with a cosmic string is naturally seen as a topological defect having a chiral nature as well as a space-like helical structure, with delta function singularities present in both the scalar curvature and torsion, characterizing the geometry of the spacetime [37,38]. Note that although the authors in Ref. [37] used the terminology ‘cosmic dislocation’ to refer to this chiral cosmic string, we will follow the same terminology coined by the authors in Ref. [39] who called this combination as ‘cosmic dispiration’. It seems more appropriate due to the singularity associated to the screw dislocation aspect of the defect. Also, the spacetime produced by a cylindrically symmetric gravitational wave has a large-distance asymptotic behaviour with the same structure as the spacetime generated by a cosmic dispiration [40,41].

In fact, the presence of a torsion in the spacetime has attracted a great deal of attention, for instance, due to interaction with fermions [42–46]. A line of research dealing with torsion as a topological defect in crystalline solids with the use of differential geometry can be found in Refs. [47,48]. Examples of these defects associated with torsion are the screw dislocations and the spiral dislocations [49]. Several studies have explored the influence of these kind of topological defects on quantum systems, such as the quantum scattering [50], the Aharonov-Bohm effect for bound states [51], geometric quantum phases [52] and geometric quantum computation [53]. The influence of torsion has also attracted attention in condensed matter system described by the Dirac equation, for instance, graphene [54–56]. Investigations about spacetimes generated by chiral cosmic strings, obtained as a solution of the Einstein-Cartan equations, can be found in Refs. [36–38,57]. In this sense, torsion effects on relativistic quantum systems associated with the presence of these chiral cosmic strings have also been reported in the literature. Among them, it is worth citing the Aharonov-Bohm effect for bound states [57] and the studies of the Dirac oscillator [58] and a relativistic position-dependent mass system [59].

In Ref. [37], the authors pointed it out that a cosmic dispiration produces lensing effects both in its transversal and longitudinal directions, in contrast with the cosmic string case where only the lensing effect in its transversal direction exists. In this sense, it is possible to distinguish the usual cosmic string from the chiral space-like cosmic string, that is, the cosmic dispiration.

In particular, only a few studies about cosmic dispiration spacetime, in the framework of quantum vacuum fluctuations, have been reported in the literature [39,60,61]. In Ref. [60] the authors performed the investigation of the scalar Casimir energy density in the spacetime of a

screw dislocation using the zeta function technique and obtained only a long-range approximated expression. On the other hand, the authors in Ref. [39] developed an investigation of quantum fluctuations of the massless scalar field in the spacetime of a cosmic dispiration and calculated the propagator and an approximated expression for the vacuum expectation value (VEV) of the energy-momentum tensor. In this paper, we present an exact expression for the VEV of the energy-momentum tensor arising from quantum vacuum fluctuations of both the massive and massless scalar fields in a high-dimensional cosmic dispiration spacetime.

This paper is organized as follows. In Section 2 we introduce the spacetime of a cosmic dispiration and calculate the two-point function, namely, the Wightman function. Section 3 is devoted for the complete calculation of the expressions for the VEV of the field squared in both cases, the massive and massless scalar fields. Using results from the two previous Sections, in Section 4 we calculate the VEV of the energy-momentum tensor. Finally, in Section 5 we present our final remarks. We have also dedicated an Appendix to obtain important expressions used in the development of our calculation of both the Wightman function and the VEV of the energy-momentum tensor. Throughout the paper we use natural units $G = \hbar = c = 1$.

2 Klein-Gordon equation and Wightman function

In this section, we consider a spacetime with nontrivial topology that arises due to the combined effects of a cosmic string and a screw dislocation, that is, the spacetime of a cosmic dispiration. The solution of the Klein-Gordon equation is taken only assuming that the radial solution is regular at the origin. This solution allows us to investigate the effect of the nontrivial topology of the cosmic dispiration spacetime on the VEV of physical observables.

Let us start by considering the following metric that describes an idealized $(D+1)$ -dimensional cosmic dispiration spacetime in cylindrical coordinates:

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = dt^2 - dr^2 - r^2 d\varphi^2 - (dz + \kappa d\varphi)^2 - \sum_{i=4}^D (dx^i)^2, \quad (2.1)$$

where κ is a constant parameter associated with the screw dislocation¹, $D \geq 3$ and $(r, \phi, z, x^4, \dots, x^D)$ are the generalized cylindrical coordinates taking values in the ranges $r \geq 0$, $0 \leq \varphi \leq \varphi_0 = 2\pi/q$ and $-\infty < (t, z, x^i) < +\infty$, for $i = 4, \dots, D$. The parameter $q \geq 1$ codifies the presence of the cosmic string which we assume to be on the $(D-2)$ -dimensional hypersurface $r = 0$. This parameter, in $D = 3$, is associated with the linear mass density of the string, μ_0 , through the relation $q^{-1} = 1 - 4G\mu_0$, where G is the Newton's gravitational constant. The usual identification condition for the spatial points are $(r, \varphi, z, x^4, \dots, x^D) \rightarrow (r, \varphi + 2\pi/q, z, x^4, \dots, x^D)$.

The line element (2.1) is locally flat and can be written in a Minkowskian form if we define the new spatial coordinate $Z = z + \kappa\varphi$, providing

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = dt^2 - dr^2 - r^2 d\varphi^2 - dZ^2 - \sum_{i=4}^D (dx^i)^2, \quad (2.2)$$

where the new identification condition is found to be

$$(r, \varphi, Z, x^4, \dots, x^D) \rightarrow (r, \varphi + 2\pi/q, Z + 2\pi\kappa/q, x^4, \dots, x^D). \quad (2.3)$$

¹The constant κ is actually called burgers vector in the framework of theories of continuum media [36]. In the context of the Einstein-Cartan theory of gravity it is identified with $\frac{2GJ^z}{\pi}$, where J^z is a component of a two-dimensional vector tangent to the string worldsheet (see Refs. [37, 38] for more details).

The Klein-Gordon equation for a nonminimally coupled scalar field, $\Phi(x)$, can be written in the form

$$\left[\frac{1}{\sqrt{|g|}} \partial_\mu \left(\sqrt{|g|} g^{\mu\nu} \partial_\nu \right) + m^2 + \xi R \right] \Phi(x) = 0, \quad (2.4)$$

where ξ is the curvature coupling constant to gravity, R is the Ricci scalar and indices $j, l = 1, 2, 3, \dots, D$ describe the spatial coordinates. In addition, as we are considering an idealized cosmic dispiration spacetime we have that $R = 0$ for $r \neq 0$.

Using the line element (2.1), we can further write Eq. (2.4) in the form

$$\left[\frac{\partial^2}{\partial t^2} - \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) - \frac{1}{r^2} \left(\frac{\partial}{\partial \varphi} - \kappa \frac{\partial}{\partial z} \right)^2 - \frac{\partial^2}{\partial z^2} - \sum_{i=4}^D \frac{\partial^2}{\partial x^{i2}} + m^2 \right] \Phi(x) = 0, \quad (2.5)$$

which has the general solution

$$\Phi_k(x) = C R(r) e^{-i\omega_k t + inq\varphi + i\nu z + i\mathbf{p} \cdot \mathbf{r}_\parallel}, \quad (2.6)$$

where C is a normalization constant and \mathbf{r}_\parallel and \mathbf{p} represent, respectively, the coordinates of the extra dimensions and their corresponding momenta. Substituting Eq. (2.6) into (2.5) we get the following differential equation

$$\left[\frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} + \eta^2 - \frac{(qn - \kappa\nu)^2}{r^2} \right] R(r) = 0, \quad (2.7)$$

where $\omega_k^2 = m^2 + \eta^2 + \nu^2 + p^2$ and $k = (\eta, n, \nu, p)$ is the set of quantum numbers. The above equation is a Bessel differential equation whose solution is a combination of the Bessel $J_\mu(x)$ and Neumann $N_\mu(x)$ functions. Since the order of these functions depends on the continuum quantum number ν , the Neumann function, in general, cannot be square-integrable at origin. Thus, we have to consider only the regular solution at origin of Eq. (2.7), that is,

$$R(r) = J_{|qn - \kappa\nu|}(\eta r). \quad (2.8)$$

The normalization constant C can be calculated using the orthonormalization condition

$$\int d^D x \sqrt{|g|} \Phi_k(x) \Phi_{k'}^*(x) = \frac{1}{2\omega_k} \delta_{k,k'}, \quad (2.9)$$

where the delta symbol on the right-hand side is understood as Kronecker delta for n and Dirac delta function for the continuous quantum numbers, η, ν and \mathbf{p} . Using Eqs. (2.6) and (2.8), the orthonormalization condition in Eq. (2.9) provides

$$|C|^2 = \frac{q\eta}{2\omega_k (2\pi)^{D-1}}. \quad (2.10)$$

Therefore, the final form for the solution (2.6) is given by

$$\Phi_k(x) = \left[\frac{q\eta}{2\omega_k (2\pi)^{D-1}} \right]^{\frac{1}{2}} J_{|qn - \kappa\nu|}(\eta r) e^{-i\omega_k t + inq\varphi + i\nu z + i\mathbf{p} \cdot \mathbf{r}_\parallel}. \quad (2.11)$$

Equivalently, in terms of the the coordinates $(t, r, \varphi, Z, x^4, \dots, x^D)$, the general solution obeying the identification condition (2.3) is given by

$$\Phi_k(x) = \left[\frac{q\eta}{2\omega_k (2\pi)^{D-1}} \right]^{\frac{1}{2}} J_{|qn - \kappa\nu|}(\eta r) e^{-i\omega_k t + i(nq - \kappa\nu)\varphi + i\nu Z + i\mathbf{p} \cdot \mathbf{r}_\parallel}. \quad (2.12)$$

Note that although the solutions (2.11) and (2.12) lead to the same physical results, it is more convenient to work with the latter one since the line element (2.2) allows us, as we will see, to write the energy-momentum tensor in a simpler form.

The topology of the spacetime described by the line element (2.2) along with the requirement that the radial solution (2.8) be regular at the origin change the spectrum of vacuum fluctuations when compared with the Minkowski spacetime. Thus, in order to study the corresponding changes in the VEV of physical observables it is necessary to describe the properties of the vacuum state, $|0\rangle$, through the positive frequency Wightman function, $W(x, x') = \langle 0 | \Phi(x) \Phi(x') | 0 \rangle$, being $\Phi(x)$ the field operator. The latter can be expanded in terms of the complete set of normalized mode functions, $\{\Phi_k(x), \Phi_k^*(x)\}$, given by Eq. (2.11)-(2.12). Consequently, we obtain

$$W(x, x') = \sum_k \Phi_k(x) \Phi_k^*(x'), \quad (2.13)$$

with the compact notation

$$\sum_k = \int dp^{(D-3)} \int_{-\infty}^{\infty} d\nu \int_0^{\infty} d\eta \sum_{n=-\infty}^{\infty}. \quad (2.14)$$

The detailed calculation for the Wightman function is performed in the Appendix A and the final result is given by

$$\begin{aligned} W(x, x') &= \frac{m^{(D-1)}}{(2\pi)^{\frac{(D+1)}{2}}} \left[\sum_l f_{\frac{D-1}{2}}(m\sigma_l) \right. \\ &\quad \left. - \frac{q}{\pi^2} \sum_{n=-\infty}^{\infty} \int_0^{\infty} dy f_{\frac{D-1}{2}}(m\sigma_{y,n}) M_{n,q}(\Delta\varphi, y) \right], \end{aligned} \quad (2.15)$$

where the discrete index l obeys the condition

$$-\frac{q}{2} + \frac{\Delta\varphi}{\varphi_0} \leq l \leq \frac{q}{2} + \frac{\Delta\varphi}{\varphi_0}, \quad (2.16)$$

and we have used, in terms of the Macdonald function $K_\mu(x)$, the notation

$$f_\mu(x) = \frac{K_\mu(x)}{x^\mu}. \quad (2.17)$$

Note also that

$$\begin{aligned} \sigma_l^2 &= \left[\Delta\zeta^2 - 2rr' \cos(2\pi l/q - \Delta\varphi) + (\Delta Z - \bar{\kappa}l)^2 \right], \\ \sigma_{y,n}^2 &= \left[\Delta\zeta^2 + 2rr' \cosh(y) + (\Delta Z + \bar{\kappa}n)^2 \right], \end{aligned} \quad (2.18)$$

with $\Delta\zeta^2 = \Delta\tau^2 + \Delta\mathbf{r}_\parallel^2 + r^2 + r'^2$ and $\bar{\kappa} = \kappa/\varphi_0$. Additionally, the function $M_{n,q}$ is also presented in Appendix A and is written here as

$$M_{n,q}(\Delta\varphi, y) = \frac{1}{2} \left\{ \frac{\left(\frac{q}{2} + \frac{\Delta\varphi}{\varphi_0} + n\right)}{\left(\frac{q}{2} + \frac{\Delta\varphi}{\varphi_0} + n\right)^2 + \left(\frac{y}{\varphi_0}\right)^2} - \frac{\left(-\frac{q}{2} + \frac{\Delta\varphi}{\varphi_0} + n\right)}{\left(-\frac{q}{2} + \frac{\Delta\varphi}{\varphi_0} + n\right)^2 + \left(\frac{y}{\varphi_0}\right)^2} \right\}. \quad (2.19)$$

On the other hand, by taking the limit $m \rightarrow 0$ in (2.15), the Wightman function for the massless case is found to be

$$\begin{aligned} W(x, x') &= \frac{2^{\frac{(D-1)}{2}} \Gamma\left(\frac{D-1}{2}\right)}{2(2\pi)^{\frac{(D+1)}{2}}} \left[\sum_l \frac{1}{\sigma_l^{(D-1)}} \right. \\ &\quad \left. - \frac{q}{\pi^2} \sum_{n=-\infty}^{\infty} \int_0^{\infty} dy \frac{1}{\sigma_{y,n}^{(D-1)}} M_{n,q}(\Delta\varphi, y) \right]. \end{aligned} \quad (2.20)$$

Note that for $1 \leq q < 2$ the first term on the r.h.s of Eqs. (2.15) and (2.20) is absent. For these values of q the Wightman function for the massless scalar field above is in agreement, for $D = 3$, with the propagator obtained in Ref. [39]². However, here we have provided closed expressions for all values of q , Eqs. (2.15) and (2.20), for massive and massless scalar fields in a high-dimensional cosmic dispiration spacetime, respectively.

3 Vacuum expectation value of the field squared

The VEV of the field squared, $\langle \Phi^2 \rangle$, is formally obtained from the Wightman functions (2.15) in the coincidence limit $x' \rightarrow x$, i.e.,

$$\langle \Phi^2 \rangle = \lim_{x' \rightarrow x} W(x, x'). \quad (3.1)$$

However, the Wightman function (2.15) is divergent in this limit. Its divergent contribution is given by the Hadamard function obtained from the $l = 0$ term and, according to the standard prescription discussed in Refs. [62, 63], it must be subtracted from the Wightman function $W(x, x')$. So, the renormalized result of the above VEV is

$$\langle \Phi^2 \rangle_{\text{ren}} = \lim_{x' \rightarrow x} [W(x, x') - G_{\text{H}}(x, x')], \quad (3.2)$$

where the Hadamard function $G_{\text{H}}(x, x')$ is given by

$$G_{\text{H}}(x, x') = \frac{m^{(D-1)}}{(2\pi)^{\frac{(D+1)}{2}}} f_{\frac{D-1}{2}}(m\sigma_0), \quad (3.3)$$

with σ_0 defined as in (2.18) for $l = 0$, i.e.,

$$\sigma_0^2 = -\Delta t^2 + \Delta \mathbf{r}_{\parallel}^2 + r'^2 + r^2 - 2rr' \cos(\Delta\varphi) + \Delta Z^2. \quad (3.4)$$

Here we would like to call attention to the fact that the Hadamard function above is the solution of the corresponding homogeneous differential equation in the coordinate system defined by (2.2), when $q = 1$.

The renormalized VEV of the field squared (3.2) is, therefore, found to be

$$\begin{aligned} \langle \Phi^2 \rangle_{\text{ren}} &= \frac{2m^{(D-1)}}{(2\pi)^{\frac{(D+1)}{2}}} \left[\sum'_{l=1}^{[q/2]} f_{\frac{D-1}{2}} \left(m \sqrt{(\bar{\kappa}l)^2 + 4r^2 s_l^2} \right) \right. \\ &\quad \left. - \frac{q}{\pi^2} \sum_{n=-\infty}^{\infty} \int_0^{\infty} dy f_{\frac{D-1}{2}} \left(m \sqrt{(\bar{\kappa}n)^2 + 4r^2 \cosh^2(y)} \right) M_{n,q}(2y) \right], \end{aligned} \quad (3.5)$$

where $M_{n,q}(0, 2y) = M_{n,q}(2y)$ and $[q/2]$ represents the integer part of $q/2$, and the prime on the sign of the summation means that in the case $q/2$ is integer the term $l = q/2$ should be taken with the coefficient $1/2$. Note that we have also made the change, $y \rightarrow 2y$, and defined

$$s_l = \sin(\pi l/q). \quad (3.6)$$

Some limiting cases of Eq. (3.5) are possible to be considered for $D = 3$. For instance, for fixed $m\bar{\kappa}$, in the limit $mr \gg 1$, we can ignore the summation indices l and n in the argument

²Note that the authors in Ref. [39] calculated the Feynman propagator. Thus, our result in (2.20) coincides with theirs as long as we use the relation $W(x, x') = iG^{(\alpha, \kappa)}(x, x')$, where $G^{(\alpha, \kappa)}(x, x')$ is the Feynman propagator in their notation.

of the $f_\mu(x)$ functions. This procedure can only be adopted on the second term on the r.h.s of Eq. (3.5) because the function $M_{n,q}(2y)$ presents a negligible contribution for large values of n . Thereby, this provides, as a leading contribution, the expression of the renormalized VEV of the field squared in the pure cosmic string spacetime. In other words, far away from the cosmic dispiration the screw dislocation is negligible. On the other hand, keeping mr fixed, the leading contribution in the limit $m\bar{\kappa} \gg 1$ comes from $n = 0$ in the second term on the r.h.s of (3.5). For the first term, we can neglect the term with s_l inside the brackets. Thus, in this regime we obtain the following approximation:

$$\begin{aligned}\langle \Phi^2 \rangle_{\text{ren}} &\simeq \frac{m^2}{2\pi^2} \left[\sqrt{\frac{\pi}{2}} \frac{e^{-m\bar{\kappa}}}{(m\bar{\kappa})^{\frac{3}{2}}} - \frac{1}{mr} \int_0^\infty dy \frac{K_1(2mr \cosh y)}{\cosh y} \frac{1}{(\pi^2 + 4y^2)} \right] \\ &\simeq \frac{m}{2\pi^2 r} \int_0^\infty dy \frac{K_1(2mr \cosh y)}{\cosh y} \frac{1}{(\pi^2 + 4y^2)}.\end{aligned}\quad (3.7)$$

Note that, in this approximation, the integral term is the dominant one since the first term is exponentially suppressed. As a consequence, the final result for Eq. (3.7) is independent of κ .

Additionally, in the limit $m \rightarrow 0$, Eq. (3.5) leads to the expression of the renormalized VEV of the massless scalar field squared, i.e,

$$\begin{aligned}\langle \Phi^2 \rangle_{\text{ren}} &= \frac{\Gamma\left(\frac{D-1}{2}\right)}{2\pi^{\frac{(D+1)}{2}} \bar{\kappa}^{D-1}} \left[\sum_{l=1}^{[q/2]'} [l^2 + \epsilon^2 s_l^2]^{\frac{1-D}{2}} \right. \\ &\quad \left. - \frac{q}{\pi^2} \sum_{n=-\infty}^\infty \int_0^\infty dy [n^2 + \epsilon^2 \cosh^2(y)]^{\frac{1-D}{2}} M_{n,q}(2y) \right],\end{aligned}\quad (3.8)$$

where $\epsilon = 2r/\bar{\kappa} = qr/\pi\kappa$. The result in (3.8) for the VEV of the field squared in the massless case, coincides, for $D = 3$, with the one obtained in Ref. [39] when $q < 2$. However we would like to emphasize that we have obtained, for all possible values of q , closed general expressions for the Wightman function (2.15) and, consequently, for the the renormalized VEV of the field squared (3.2) for the massive case, in a high-dimensional cosmic dispiration spacetime. We have also formally demonstrated in the Appendix B that when $\kappa = 0$ we recover the Wightman function in the pure cosmic string spacetime and all the resulting expressions.

For the three-dimensional case, $D = 3$, the summation in n present in Eq. (3.8) provides a closed expression and we find that the renormalized VEV of the massless field squared is written as

$$\langle \Phi^2 \rangle_{\text{ren}} = \frac{1}{2\pi^2 \bar{\kappa}^2} \left[\sum_{l=1}^{[q/2]'} [l^2 + \epsilon^2 s_l^2]^{-1} - \frac{q}{\pi^2} \int_0^\infty g(y, \epsilon, q) dy \right], \quad (3.9)$$

where

$$\begin{aligned}g(y, \epsilon, q) &= \frac{2\pi^3}{\epsilon f(y, q) \cosh(y)} \left\{ q [4\epsilon^2 \pi^2 \cosh^2(y) + q^2(\pi^2 + 4y^2)] \coth(\epsilon\pi \cosh(y)) \right. \\ &\quad \left. + \frac{2\epsilon \cosh(y) [4\epsilon^2 \pi^2 \cosh^2(y) + q^2(\pi^2 - 4y^2)] \sin(q\pi) - 8q^2 \epsilon \pi y \cosh(y) \sinh(2qy)}{\cosh(2qy) - \cos(\pi q)} \right\},\end{aligned}\quad (3.10)$$

with

$$f(y, q) = 16\epsilon^4 \pi^4 \cosh^4(y) + 8q^2 \epsilon^2 \pi^2 (\pi^2 - 4y^2) \cosh^2(y) + q^4 (\pi^2 + 4y^2)^2. \quad (3.11)$$

Let us now investigate the limiting cases $\epsilon \ll 1$ and $\epsilon \gg 1$ of Eq. (3.9). Thereby, in the limit $\epsilon \gg 1$, we recover the result for the VEV of the field squared, in the massless case, in the pure cosmic string spacetime [64, 65] using the expression

$$\lim_{\epsilon \rightarrow \infty} \epsilon^2 g(y, \epsilon, q) = \frac{\pi}{\cosh^2(y)} \frac{\sin(q\pi)}{[\cosh(2qy) - \cos(\pi q)]}. \quad (3.12)$$

In other words, the screw dislocation effects are negligible in the regime where $r \gg \pi\kappa/q$.

On the other hand, in the opposite limit, when $\epsilon \ll 1$, one can expand Eq. (3.10) in power series for small ϵ . In this case, the main contribution for the renormalized VEV of the massless field squared, Eq. (3.9), is found to be

$$\langle \Phi^2 \rangle_{\text{ren}} \simeq -\frac{1}{48\pi^2 r^2} g_0 + O(\epsilon^2) \quad (3.13)$$

where

$$g_0 \simeq 12 \int_0^\infty dy \frac{1}{(\pi^2 + 4y^2)} \frac{1}{\cosh^2(y)} \simeq 1, \quad (3.14)$$

is obtained numerically.

We would like now to call attention to a very interesting fact that has already been pointed out by the authors in Ref. [39]. For this purpose, let us consider $q < 2$ and $\kappa = 0$, where in this case the first term on the r.h.s of (3.8) is absent. The VEV of the field squared in the cosmic string spacetime is then given by

$$\langle \Phi^2 \rangle_{\text{ren}} = -\frac{q \sin(q\pi)}{8\pi^3 r^2} \int_0^\infty dy \frac{1}{[\cosh(2yq) - \cos(q\pi)]} \frac{1}{\cosh^2(y)}. \quad (3.15)$$

By noting that the main contribution for the expression above comes from small values of y , we can obtain an approximation for $q \ll 1$ as

$$\begin{aligned} \langle \Phi^2 \rangle_{\text{ren}} &\simeq -\frac{1}{4\pi^2 r^2} \int_0^\infty dy \frac{1}{(\pi^2 + 4y^2)} \frac{1}{\cosh^2(y)} \\ &\simeq -\frac{1}{48\pi^2 r^2} g_0. \end{aligned} \quad (3.16)$$

Thereby, we can see that the approximation for the expression (3.15) when $q \ll 1$ gives, as the main contribution, the expression in (3.16). The latter is equivalent to the leading contribution in Eq. (3.13) obtained when $\kappa \neq 0$ and $\epsilon \ll 1$. This can also be verified through the alternative analytic expression, given by Eq. (14) in Ref. [39], for the VEV of the field squared in the cosmic string spacetime.

4 Energy-momentum tensor

Since we have obtained the Wightman function (2.15) and the the renormalized VEV of the field squared, Eq. (3.5), we are now in position to calculate the VEV of the energy-momentum tensor in the high-dimensional cosmic dispiration spacetime. By adopting the coordinate system in which the metric (2.2) is written, the components of the energy-momentum tensor can be calculated by using the formula [65]

$$\langle T_{\mu\nu} \rangle = \lim_{x' \rightarrow x} \partial_{\mu'} \partial_{\nu} W(x, x') + [(\xi - 1/4) g_{\mu\nu} \square - \xi \nabla_{\mu} \nabla_{\nu} - \xi R_{\mu\nu}] \langle \Phi^2 \rangle, \quad (4.1)$$

where, in the spacetime under consideration, for $r > 0$, the Ricci tensor $R_{\mu\nu}$ vanishes. Thus, using Eqs. (2.15) and (3.5) it is possible to calculate the renormalized VEV of the energy-momentum tensor as we shall see below.

Let us then start by calculating, $\square \langle \Phi^2 \rangle_{\text{ren}}$. In this case the only derivative contribution in the d'Alembertian operator is the one due to the radial component, r . So, we have

$$\begin{aligned} \square \langle \Phi^2 \rangle_{\text{ren}} &= -\frac{8m^{(D+1)}}{(2\pi)^{\frac{(D+1)}{2}}} \left\{ \sum_{l=1}^{[q/2]'} \left[(2mr)^2 s_l^4 f_{\frac{D+3}{2}}(\lambda_l) - 2s_l^2 f_{\frac{D+1}{2}}(\lambda_l) \right] \right. \\ &\quad \left. - \frac{q}{\pi^2} \sum_{n=-\infty}^{\infty} \int_0^{\infty} dy \left[(2mr)^2 \cosh^4(y) f_{\frac{D+3}{2}}(\lambda_n) - 2 \cosh^2(y) f_{\frac{D+1}{2}}(\lambda_n) \right] M_{n,q}(2y) \right\}. \end{aligned} \quad (4.2)$$

Moreover, it is also necessary to calculate the ordinary derivative of $W(x, x')$ with respect to

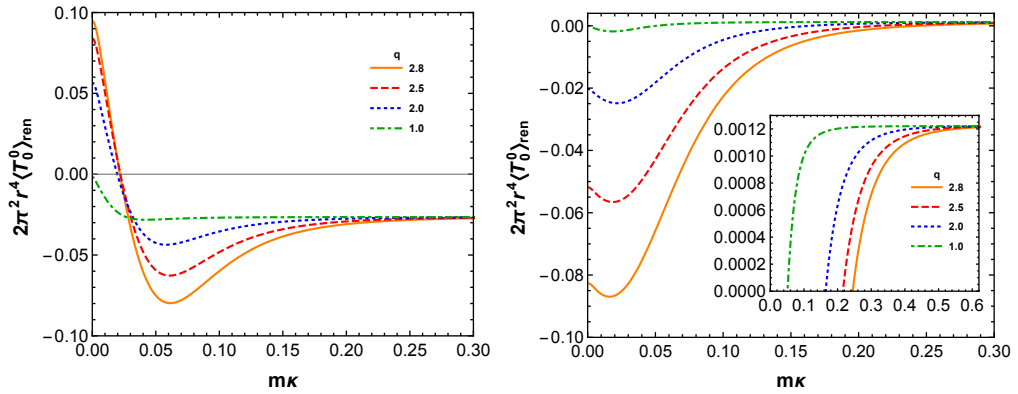


Figure 1: For $D = 3$, the $(0,0)$ -component of the energy-momentum tensor, $\langle T_0^0 \rangle_{\text{ren}}$, multiplied by $2\pi^2 r^4$, in the massive scalar field case, is plotted as a function of $m\kappa$ for different values of the cosmic string parameter, q , considering $mr = 0.1$. The plot on the left is for $\xi = 0$ whilst the one on the right is for $\xi = \frac{1}{6}$. Also, the smaller plot on the right indicates that the curves do not go to zero as they seem to do in the larger plot.

the spacetime coordinates. Thus, taking the Wightman function (2.15), we obtain after long and intermediate steps the renormalized VEV of the energy-momentum tensor for the massive scalar field

$$\begin{aligned} \langle T_\nu^\mu \rangle_{\text{ren}} &= \frac{2m^{(D+1)}}{(2\pi)^{\frac{(D+1)}{2}}} \left[\sum_{l=1}^{[q/2]'} F_{\nu,l}^\mu(2mr, m\bar{\kappa}, s_l) \right. \\ &\quad \left. - \frac{q}{\pi^2} \sum_{n=-\infty}^{\infty} \int_0^{\infty} dy F_{\nu,n}^\mu(2mr, m\bar{\kappa}, \cosh y) M_{n,q}(2y) \right], \end{aligned} \quad (4.3)$$

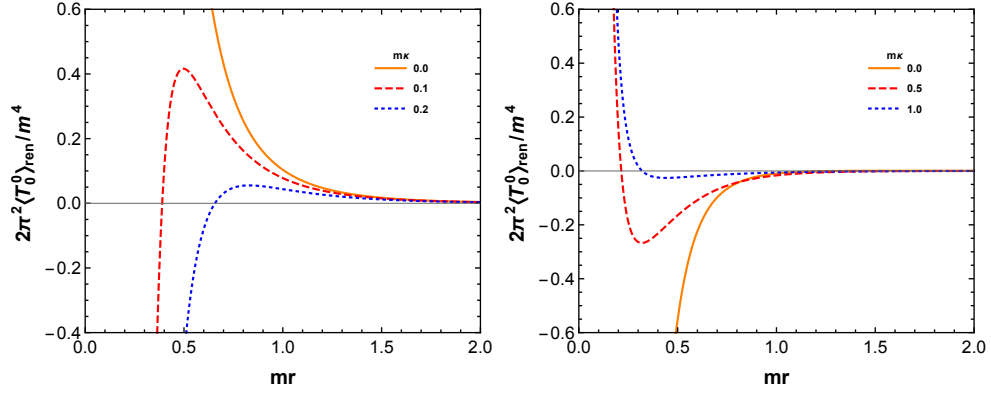


Figure 2: For $D = 3$, the $(0,0)$ -component of the energy-momentum tensor, $\langle T_0^0 \rangle_{\text{ren}}$, in units of $m^4/2\pi^2$, in the massive scalar field case, is plotted as a function of mr for different values of $m\kappa$, considering $q = 2.5$. The plot on the left is for $\xi = 0$ whilst the one on the right is for $\xi = \frac{1}{6}$.

where the functions $F_{\nu,\sigma}^\mu(u, g, v)$ are defined as

$$\begin{aligned}
F_{0,\sigma}^0(u, g, v) &= u^2 v^4 (1 - 4\xi) f_{\frac{D+3}{2}}(\lambda_\sigma) - [2v^2(1 - 4\xi) + 1] f_{\frac{D+1}{2}}(\lambda_\sigma), \\
F_{1,\sigma}^1(u, g, v) &= (4\xi v^2 - 1) f_{\frac{D+1}{2}}(\lambda_\sigma), \\
F_{2,\sigma}^2(u, g, v) &= (1 - 4\xi v^2) \left[u^2 v^2 f_{\frac{D+3}{2}}(\lambda_\sigma) - f_{\frac{D+1}{2}}(\lambda_\sigma) \right], \\
F_{3,\sigma}^3(u, g, v) &= F_{0,\sigma}^0(u, g, v) + (\bar{\kappa} m \sigma)^2 f_{\frac{D+3}{2}}(\lambda_\sigma), \\
F_{2,\sigma}^3(u, g, v) &= \frac{1}{4} \bar{\kappa} \sigma \sin(2\pi\sigma/q) u^2 f_{\frac{D+3}{2}}(\lambda_\sigma), \\
F_{i,\sigma}^i(u, g, v) &= F_{0,\sigma}^0(u, g, v), \quad \text{for } i = 4, \dots, D,
\end{aligned} \tag{4.4}$$

with

$$\lambda_\sigma = \sqrt{u^2 v^2 + (g\sigma)^2}. \tag{4.5}$$

In (4.4) and (4.5) the greek letter σ stands for l in the first term in (4.3) and for n in the second

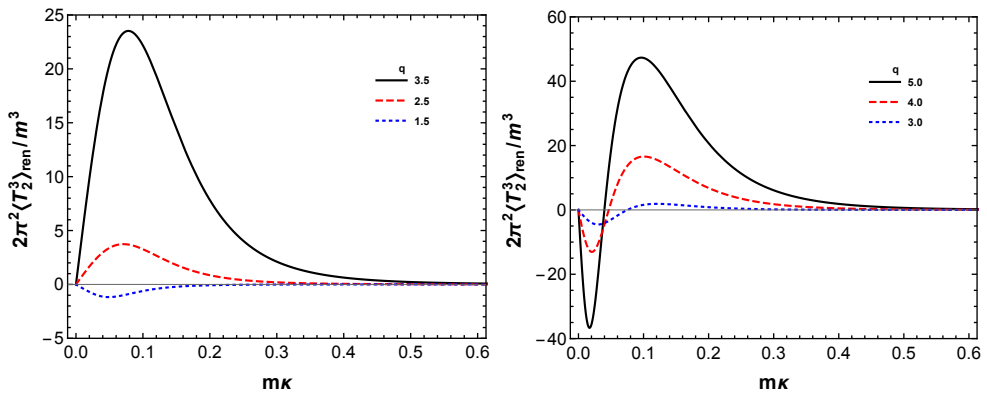


Figure 3: For $D = 3$, the off-diagonal component of the energy-momentum tensor, $\langle T_2^3 \rangle_{\text{ren}}$, in units of $m^3/2\pi^2$, in the massive scalar field case, is plotted as a function of $m\kappa$ for different values of q , considering $mr = 0.2$.

term. Also, one can see that the off-diagonal component is independent of the coupling constant

ξ . The boost invariance in the directions x^i , $i = 4, \dots, D$ provides us with (no summation over i) $F_{i,\sigma}^i(u, g, v) = F_{0,\sigma}^0(u, g, v)$. On the other hand, because the boost invariance in the z -direction is broken by the presence of the torsion, we have that $F_{3,\sigma}^3(u, g, v) \neq F_{0,\sigma}^0(u, g, v)$, as we can see in Eq. (4.4). The latter also shows an off-diagonal component which only exists if both defects, screw dislocation and cosmic string, are present otherwise it vanishes. Note that in order to calculate the $(2, 2)$ -component of the energy-momentum tensor we used Eq. (52) from Ref. [64]. We also made use of Eq. (A.11) from Ref. [29] to calculate the single non-diagonal $(3, 2)$ -component following the same procedure used in Appendix A.

In Fig.1 we have plotted, for $D = 3$, the $(0, 0)$ -component of the energy-momentum tensor multiplied by $2\pi^2 r^4$, with respect to $m\kappa$ for different values of q and taking $mr = 0.1$, as indicated. The plot on the left is for the minimally coupled massive scalar field whilst the plot on the right is for the conformally coupled one. In Fig.2, for $D = 3$, one can see the plot of $\langle T_0^0 \rangle_{\text{ren}}$, in units of $m^4/2\pi^2$, with respect to mr for different values of $m\kappa$ and taking $q = 2.5$. Again, the plot on the left is for $\xi = 0$ whilst the plot on the right is for $\xi = 1/6$. It is clear that, compared to $m\kappa = 0$, the presence of the torsion changes the behaviour of $\langle T_0^0 \rangle_{\text{ren}}$ near the string significantly. Furthermore, for $D = 3$, the plot of the off-diagonal component $\langle T_2^3 \rangle_{\text{ren}}$, in units of $m^3/2\pi^2$, with respect to $m\kappa$ is shown in Fig.3. The curves are obtained for different values of q and taking $mr = 0.2$.

We have proved that the energy-momentum tensor (4.3) satisfies two important requirements: the covariant conservation condition $\nabla_\mu \langle T_\nu^\mu \rangle_{\text{ren}} = 0$, which can be reduced down to the relation

$$\langle T_2^2 \rangle_{\text{ren}} = \partial_r (r \langle T_1^1 \rangle_{\text{ren}}), \quad (4.6)$$

as well as the trace identity

$$\langle T_\mu^\mu \rangle_{\text{ren}} = D(\xi - \xi_D) \nabla_\mu \nabla^\mu \langle \Phi^2 \rangle_{\text{ren}} + m^2 \langle \Phi^2 \rangle_{\text{ren}}. \quad (4.7)$$

Now we want to consider the asymptotic behaviours of both the $(0, 0)$ and the off-diagonal components of Eq. (4.3), for $D = 3$, in the regimes $m\bar{\kappa} \gg mr$ and $mr \gg m\bar{\kappa}$. In the latter, based on the same argument we used for the VEV of the field squared, (3.5), the leading contribution to the $(0, 0)$ -component of the energy-momentum tensor coincides with the corresponding one in the cosmic string spacetime [64, 65]. As to the regime where $m\kappa \gg mr$, also assuming $mr \ll 1$, we find

$$\begin{aligned} \langle T_0^0 \rangle_{\text{ren}} &\simeq \frac{m^4}{2\pi^2} \left[\sum_{l=1}^{[q/2]'} F_{0,l}^0(m\bar{\kappa}l) - \frac{1}{4m^4 r^4} \int_0^\infty dy \frac{[2 \cosh^2(y)(1 - 4\xi) - 1]}{\cosh^4(y)(\pi^2 + 4y^2)} \right] \\ &\simeq \frac{m^4}{2\pi^2} \sum_{l=1}^{[q/2]'} F_{0,l}^0(m\bar{\kappa}l) - \frac{1}{8\pi^2 r^4} \left[\frac{(1 - 4\xi)}{6} - 0.06 \right], \end{aligned} \quad (4.8)$$

where $F_{0,l}^0(2mr, m\bar{\kappa}, s_l) = F_{0,l}^0(m\bar{\kappa})$ is defined as in (4.4) but with $\lambda_l \simeq m\bar{\kappa}l$. Note that we have used the small argument approximation for the Macdonald function $K_\mu(x)$ [66]. Note also that the contribution on the second term on the r.h.s of Eq. (4.8) comes from the $n = 0$ term in the sum of Eq. (4.3). Moreover, the additional requirement, $m\bar{\kappa} \gg 1$, along with the large argument approximation of $K_\mu(x)$ [66], makes the first term on the r.h.s of Eq. (4.8) to be exponentially suppressed by the factor $\frac{e^{-m\bar{\kappa}}}{(m\bar{\kappa})^{\frac{5}{2}}}$ and, as a consequence, the second term is the dominant one. This asymptotic behaviour is shown in Figs.1,2.

On the other hand, for the off-diagonal component, in the regime $mr \gg m\bar{\kappa}$, with $mr \gg 1$, we obtain

$$\langle T_2^3 \rangle_{\text{ren}} \simeq \frac{m^4 \kappa}{4\pi q (2mr)^{\frac{3}{2}}} \sqrt{\frac{\pi}{2}} \left[\frac{\sin(2\pi/q)}{\sin^{\frac{7}{2}}(\pi/q)} e^{-2mr \sin(\pi/q)} - \frac{q}{\pi^2} \int_0^\infty dy \frac{g(q, y)}{\cosh^{\frac{7}{2}}(y)} e^{-2mr \cosh y} \right], \quad (4.9)$$

where

$$g(q, y) = \sum_{n=-\infty}^{\infty} n \sin(2\pi n/q) M_{n,q}(2y). \quad (4.10)$$

The summation in n in Eq. (4.10) is possible to be performed but the resulting expression is too long; for that reason we shall not write it here. The asymptotic behaviour described by Eq. (4.9) can be seen in Fig.3.

Furthermore, in the regime where $m\bar{\kappa} \gg mr$, also with $m\bar{\kappa} \gg 1$, the off-diagonal component is given by

$$\begin{aligned} \langle T_2^3 \rangle_{\text{ren}} &= \frac{m^3}{2\pi^2} (mr)^2 \sqrt{\frac{\pi}{2}} \left[\sum_{l=1}^{[q/2]} \frac{\sin(2\pi l/q) e^{-m\bar{\kappa}l}}{(m\bar{\kappa}l)^{\frac{5}{2}}} - \frac{m\bar{\kappa}q}{\pi^2} \sum_{n=-\infty}^{\infty} n \sin(2\pi n/q) e^{-m\bar{\kappa}n} \right. \\ &\quad \times \left. \int_0^\infty dy \frac{M_{n,q}(2y)}{[(2mr)^2 \cosh^2(y) + (m\bar{\kappa}n)^2]^{\frac{7}{2}}} \right]. \end{aligned} \quad (4.11)$$

In this regime, for the values of $q > 2$, the first term is the dominant one and $\langle T_2^3 \rangle_{\text{ren}}$ goes to zero from above; on other hand for $q < 1$ the first term is absent, and we have numerically checked that decreasing the value of q the off-diagonal component above changes its sign, going from positive to negative values. This behavior is explicitly exhibited in Fig.3. One should note that, in order to get the approximation (4.11), we made use of the large argument approximation for the Macdonald function, also assuming that $e^{-\sqrt{(2mr)^2 \cosh^2(y) + (m\bar{\kappa}n)^2}} < e^{-m\bar{\kappa}n}$, for given values of the argument.

For the massless scalar field case, the VEV of the energy-momentum tensor can be obtained from Eqs. (4.3) and (4.4) in the limit $m \rightarrow 0$. This provides

$$\begin{aligned} \langle T_\nu^\mu \rangle_{\text{ren}} &= \frac{\Gamma(\frac{D+1}{2}) \epsilon^{(D+1)}}{(2r)^{D+1} \pi^{\frac{(D+1)}{2}}} \left[\sum_{l=1}^{[q/2]} \frac{H_{\nu,l}^\mu(\epsilon, s_l)}{[\epsilon^2 s_l^2 + l^2]^{\frac{D+3}{2}}} \right. \\ &\quad \left. - \frac{q}{\pi^2} \sum_{n=-\infty}^{\infty} \int_0^\infty dy \frac{H_{\nu,n}^\mu(\epsilon, \cosh y)}{[\epsilon^2 \cosh^2(y) + n^2]^{\frac{D+3}{2}}} M_{n,q}(2y) \right], \end{aligned} \quad (4.12)$$

where the new $H_{\nu,\sigma}^\mu(\epsilon, v)$ functions are defined as

$$\begin{aligned} H_{0,\sigma}^0(\epsilon, v) &= \epsilon^2 v^4 (1 - 4\xi)(D+1) - [2v^2(1 - 4\xi) + 1] (\epsilon^2 v^2 + \sigma^2), \\ H_{1,\sigma}^1(\epsilon, v) &= (4\xi v^2 - 1)(\epsilon^2 v^2 + \sigma^2), \\ H_{2,\sigma}^2(\epsilon, v) &= (1 - 4\xi v^2)(\epsilon^2 v^2 D - \sigma^2), \\ H_{3,\sigma}^3(\epsilon, v) &= H_{0,\sigma}^0(u, v) + \sigma^2(D+1), \\ H_{2,\sigma}^3(\epsilon, v) &= \frac{(D+1)}{4} \bar{\kappa} \sigma \sin(2\pi\sigma/q) \epsilon^2, \\ H_{i,\sigma}^i(\epsilon, v) &= H_{0,\sigma}^0(\epsilon, v), \quad \text{for } i = 4, \dots, D. \end{aligned} \quad (4.13)$$

As in the massive case, the boost invariance in the massless case of $\langle T_\nu^\mu \rangle_{\text{ren}}$ along the z -direction is also broken by the presence of the torsion but it is preserved for the components with $i = 4, \dots, D$. Moreover, we can promptly check that for the massless scalar field, conformally coupled to gravity, the VEV of the energy-momentum tensor (4.12) becomes traceless. One should note that in the limit $\kappa \rightarrow 0$ we recover from Eqs. (4.3) and (4.12) the expressions for

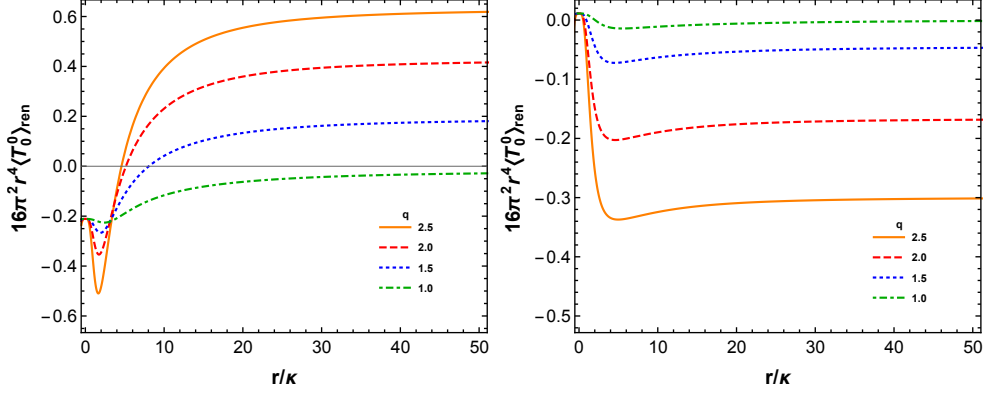


Figure 4: For $D = 3$, the (0,0)-component of the energy-momentum tensor, $\langle T_0^0 \rangle_{\text{ren}}$, multiplied by $16\pi^2 r^4$, in the massless scalar field case, is plotted as a function of r/κ for different values of the cosmic string parameter, q . The plot on the left is for $\xi = 0$ whilst the one on the right is for $\xi = \frac{1}{6}$.

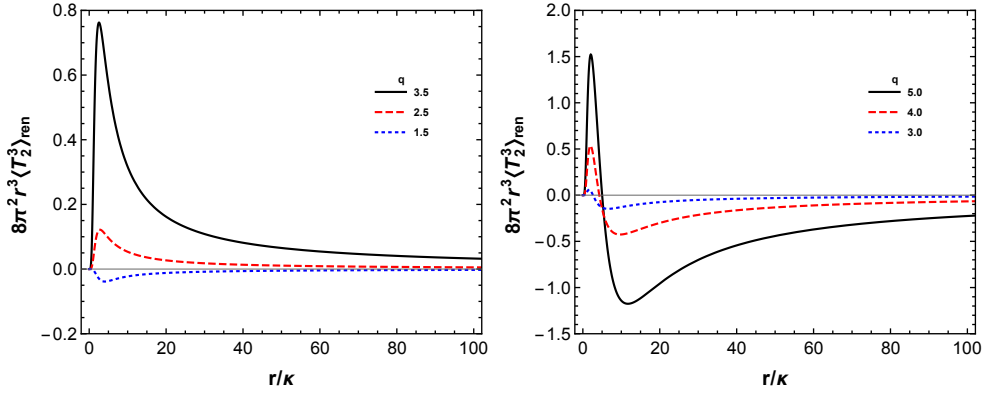


Figure 5: For $D = 3$, the off-diagonal component of the energy-momentum tensor, $\langle T_2^3 \rangle_{\text{ren}}$, multiplied by $8\pi^2 r^3$, in the massless scalar field case, is plotted as a function of r/κ for different values of q .

the energy-momentum tensor, in the massive and massless scalar fields cases, in the pure cosmic string space [64, 65]. This can essentially be done by using the relations (B.5) and (B.6) after taking $\kappa = 0$. On the other hand, by considering only the pure screw dislocation spacetime, that is, for $q = 1$, the first term on the r.h.s of each Eqs. (4.3) and (4.12) is absent and the single off-diagonal component in both cases vanishes. Note that the off-diagonal component is independent of the coupling constant ξ , as in Eq. (4.3).

In special, by writing the (0,0)-component of (4.12), for $D = 3$, in terms of derivatives with respect to ϵ^2 of the function $g(y, \epsilon, q)$, defined in (3.10), the following closed expression for the energy density for the massless scalar field is provided.

$$\begin{aligned} \langle T_0^0 \rangle_{\text{ren}} &= \frac{1}{\pi^2 \bar{\kappa}^4} \left\{ \sum_{l=1}^{[q/2]} \frac{H_{0,l}^0(\epsilon, s_l)}{[\epsilon^2 s_l^2 + l^2]^3} \right. \\ &\quad \left. - \frac{q}{\pi^2} \int_0^\infty dy \left[2\epsilon^2(1 - 4\xi)\partial_{\epsilon^2}^2 + \frac{(2\cosh^2(y)(1 - 4\xi) + 1)}{\cosh^2(y)} \partial_{\epsilon^2} \right] g(y, \epsilon, q) \right\}. \end{aligned} \quad (4.14)$$

Note that all but the off-diagonal component of the energy-momentum tensor can be written in terms of the function, $g(y, \epsilon, q)$. For the off-diagonal case it is possible to perform the sum in n

but the resulting expression is too long, so we shall not write it here.

In Fig.4 we have plotted for $D = 3$ the $(0,0)$ -component of the energy-momentum tensor multiplied by $16\pi^2 r^4$, with respect to r/κ for different values of q . The plot on the left is for $\xi = 0$ and the plot on the right is for $\xi = 1/6$. On the other hand, Fig.5 shows, for $D = 3$, the plot of the off-diagonal component $\langle T_2^3 \rangle_{\text{ren}}$, multiplied by $8\pi^2 r^3$, with respect to r/κ for different values of q .

Now we are interested in analysing, for $D = 3$, the regimes where $\epsilon \gg 1$ and $\epsilon \ll 1$ for the off-diagonal and $(0,0)$ components. For the latter, in the regime $\epsilon \ll 1$, the leading contribution comes from the second term on the r.h.s of Eq. (4.12) for $n = 0$. This contribution has already been analysed and is given by the second term on the r.h.s of Eq. (4.8). In fact, in Eq. (4.8), if we require that $m\bar{\kappa} \gg 1$, we obtain exactly the asymptotic limit $\epsilon \ll 1$ for the massless scalar field case, with the second term on the r.h.s. being the dominant one. On the other hand, in the limit $\epsilon \gg 1$, analogously to the massive case, the leader contribution is given by the energy density in $(3+1)$ -dimensional cosmic string spacetime [39, 64, 65]. This contribution is given by

$$\begin{aligned} \langle T_0^0 \rangle_{\text{ren}} &= \frac{1}{16\pi^2 r^4} \left[\sum_{l=1}^{[q/2]'} \frac{H_0^0(s_l)}{s_l^4} \right. \\ &\quad \left. - \frac{q \sin(q\pi)}{\pi} \int_0^\infty dy \frac{\cosh^{-4}(y) H_0^0(\cosh y)}{\cosh(2qy) - \cos(q\pi)} \right], \end{aligned} \quad (4.15)$$

where

$$H_0^0(v) = 2v^2(1 - 4\xi) - 1. \quad (4.16)$$

The sub-leading contribution in this case is found to be

$$\begin{aligned} \langle T_0^0 \rangle_{\text{sub}} &= \frac{q}{16\pi^2 r^4} \int_0^\infty dy \left[-\frac{9}{4}(1 - 4\xi) \cosh^{-3}(y) + \frac{3}{4} \cosh^{-5}(y) \right] \frac{\kappa}{r}, \\ &= \frac{q}{16\pi^2 r^4} \left[-\frac{9\pi}{16}(1 - 4\xi) + \frac{9\pi}{64} \right] \frac{\kappa}{r}, \\ &= \frac{9q}{256\pi r^4} \left[\frac{(16\xi - 3)}{4} \right] \frac{\kappa}{r}. \end{aligned} \quad (4.17)$$

Our leading contribution (4.15) coincides with the one obtained in Ref. [39] in the regime $\epsilon \gg 1$. The sub-leading contribution, however, is a new result and we can see that it falls off with κ/r^5 , in contrast with what Refs. [60, 61] claimed. It should be mentioned that, for $q < 2$, Ref. [39] also considered the energy-momentum tensor due to a massless scalar field in the four-dimensional cosmic dispiration spacetime, although the authors did not provide exact closed expressions. In particular, they only considered an analysis for the case $\frac{\kappa}{r} \rightarrow 0$. In this regime, their results coincide with ours except for the off-diagonal component. These asymptotics behaviour can be checked in the plots of Figs.4.

In the regime where $\epsilon \gg 1$, the off-diagonal component becomes

$$\langle T_2^3 \rangle_{\text{ren}} \simeq \frac{1}{8r^3\pi^2\epsilon} \left[\sum_{l=1}^{[q/2]'} \frac{l \sin(2\pi l/q)}{s_l^6} - \frac{q}{\pi^2} \int_0^\infty dy \frac{g(q, y)}{\cosh^6(y)} \right], \quad (4.18)$$

where the function $g(q, y)$ has been defined in Eq. (4.10). The approximation above clearly shows that the off-diagonal component, for the massless case, goes to zero with κ/r^4 , when $r \gg \bar{\kappa}$.

Regarding the regime $\epsilon \ll 1$, we can approximate the off-diagonal component as

$$\langle T_2^3 \rangle_{\text{ren}} = \frac{\epsilon^5}{8\pi^2 r^3} \bar{g}(\epsilon, q), \quad (4.19)$$

where

$$\begin{aligned} \bar{g}(\epsilon, q) &\simeq \sum_{l=1}^{[q/2]} \frac{\sin(2\pi l/q)}{l^5} \\ &- \frac{q}{\pi^2} \sum_{n=-\infty}^{\infty} \int_0^{\infty} dy \frac{n \sin(2\pi n/q)}{[\epsilon^2 \cosh^2(y) + n^2]^3} M_{n,q}(2y). \end{aligned} \quad (4.20)$$

For values of ϵ much smaller than unity the function defined above is positive for the non-integer values of $q > 2$ considered in the plot of Fig. 5. For $q = 1.5$ the first term on the r.h.s is absent and the off-diagonal component goes to zero from below. This asymptotic behaviour is shown in Fig.5. On the other hand, for integer values of $q > 2$ considered in the plot, the function $\bar{g}(\epsilon, q)$ also goes to zero from below. Although this is not clear in the plot of Fig.5, a numerical analysis is enough to verify that. Note that a numerical analysis also shows that, in the massless case, the off-diagonal component changes sign from positive to negative values as q decreases.

5 Summary and Discussion

We have considered quantum vacuum fluctuations effects that stem from the nontrivial topology of a spacetime known as cosmic dispiration whose geometry is given by Eq. (2.2), and which is a solution of the Einstein-Cartan equations. After obtaining the exact general solution (2.12) of the Klein-Gordon equation in the cosmic dispiration spacetime we were able to calculate closed expressions for the Wightman function for the massive, Eq. (2.15), and massless, Eq. (2.20), scalar fields and, as a consequence, the renormalized VEV's of the field squared in both cases were also obtained, Eqs. (3.5) and (3.8). These expressions made possible to calculate the VEV of the energy-momentum tensor presented in Eqs. (4.3) and (4.12) for the massive and massless scalar fields, respectively. In particular, in the four-dimensional cosmic dispiration spacetime, our expression for VEV of the energy-momentum tensor in the massless scalar field case shows certain agreement with the results obtained by the authors in Ref. [39], in the regime $\frac{r}{\kappa} \rightarrow \infty$ and for $1 < q < 2$. Nevertheless, we would like to emphasize that our results are more complete and general in the sense we have provided closed expressions for all quantities and for all possible values of the cosmic string parameter q , in the massive and massless scalar fields cases. Moreover, the off-diagonal component of the energy-momentum tensor is nonzero only if $q \neq 1, 2$, which can be clearly seen through Eqs. (4.4) and (4.13).

The asymptotic behaviours of the VEV's of the energy-momentum tensor in the massive and massless scalar fields cases were also discussed. These behaviours for the (0,0) and (3,2) components, for $D = 3$, are shown in the plots of Figs.1-5, including the minimally and conformally coupled cases. For instance, the asymptotic behaviour when $\frac{r}{\kappa} \gg 1$, which has the cosmic string spacetime as the leading contribution, can be confirmed through the plots in Figs.1 and 4 for the massive and massless scalar fields cases, respectively. The behaviour of the off-diagonal component are present in the plots in Figs.3 and 5. The plot in Fig.2 shows that the behaviour of the (0,0)-component of the energy-momentum tensor changes drastically near the cosmic string, in the presence of the torsion, when compared with the pure cosmic string case ($\kappa = 0$). In the massless scalar field case, we also provided the closed expression (4.14) for $\langle T_0^0 \rangle_{\text{ren}}$ in terms of the function $f(y, \epsilon, q)$, defined in Eq. (3.10). With this expression we obtained the asymptotic behaviours when $\epsilon \ll 1$ and $\epsilon \gg 1$. In special, Eq. (4.15) is shown to be the leading contribution

in the cosmic string spacetime, with the sub-leading contribution given by Eq. (4.17), which is in contrast with the ones obtained in Refs. [60,61]. The asymptotic behaviours for the off-diagonal component, in the massless case, has also been obtained and are given by Eqs. (4.18) and (4.19) when $\epsilon \gg 1$ and $\epsilon \ll 1$, respectively.

It was shown in Ref. [37] that the effect, on a quantized scalar field, of the chiral space-like cosmic string considered here is to produce a shift on the angular momentum quantum number n , which is clear if we re-examine the general solution (2.12). The latter shows that the shift is given by $n \rightarrow nq - \kappa\nu$ and, as also pointed out by the authors in [37], is equivalent to the shift produced by the coupling of a given gauge field with a charged scalar field, characterizing the famous Aharonov-Bohm effect. Thereby, in our case, the role of the charge is played by the longitudinal quantum number ν and the magnetic flux is proportional to the screw dislocation parameter, or more precisely it is given by $2\pi\kappa$. In this sense, it is possible to calculate an induced current density that stem from quantum vacuum fluctuations produced by the cosmic dispiration spacetime. As it was also pointed out in [39], this induced current density is due to a flux of longitudinal linear momentum ν . The closed expression for the Wightman function (2.15) obtained in the present paper can be used for this purpose. This work is in current preparation.

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A Calculation of the Wightman function $W(x, x')$

In order to calculate the Wightman function (2.13), we need to consider the general solution (2.12). This allows us to get

$$W(x, x') = \frac{q}{2(2\pi)^{D-1}} \sum_k \frac{e^{-i\omega_k \Delta t + i\mathbf{p} \cdot \Delta \mathbf{r}_{\parallel}}}{\omega_k} e^{i(nq - \kappa\nu)\Delta\varphi + i\nu\Delta Z} \eta J_{|qn - \kappa\nu|}(\eta r) J_{|qn - \kappa\nu|}(\eta r'), \quad (\text{A.1})$$

where $\Delta t = t - t'$, $\Delta \mathbf{r}_{\parallel} = \mathbf{r}_{\parallel} - \mathbf{r}'_{\parallel}$, $\Delta\varphi = \varphi - \varphi'$, $\Delta Z = Z - Z'$, $\omega_k^2 = m^2 + \eta^2 + \nu^2 + p^2$ and $k = (\eta, n, \nu, p)$ is the set of quantum numbers. Also, we have used the compact notation

$$\sum_k = \int dp^{(D-3)} \int_{-\infty}^{\infty} d\nu \int_0^{\infty} d\eta \sum_{n=-\infty}^{\infty}. \quad (\text{A.2})$$

Moreover, by making a Wick rotation $i\Delta t = \Delta\tau$ in Eq. (A.1) and use the relations [29]

$$\frac{e^{-\omega_k \Delta\tau}}{\omega_k} = \frac{2}{\sqrt{\pi}} \int_0^{\infty} ds e^{-s^2 \omega_k^2 - \frac{\Delta\tau^2}{4s^2}}, \quad (\text{A.3})$$

and

$$\int_0^{\infty} d\eta \eta J_{|qn - \kappa\nu|}(\eta r) J_{|qn - \kappa\nu|}(\eta r') e^{-s^2 \eta^2} = \frac{1}{2s^2} e^{-\frac{(r^2 + r'^2)}{4s^2}} I_{|qn - \kappa\nu|}(rr'/(2s^2)), \quad (\text{A.4})$$

Eq. (A.1) can be re-written as

$$W(x, x') = \frac{q\pi^{\frac{(D-4)}{2}}}{2(2\pi)^{D-1}} \int_0^{\infty} \frac{ds}{s^{D-1}} e^{-s^2 m^2 - \frac{(\Delta\zeta)^2}{4s^2}} \int_{-\infty}^{\infty} d\nu e^{-s^2 \nu^2 + i\nu\Delta z} \sum_{n=-\infty}^{\infty} e^{inq\Delta\varphi} I_{|qn - \kappa\nu|}(rr'/(2s^2)), \quad (\text{A.5})$$

where $\Delta z = \Delta Z - \kappa \Delta \varphi$ and $\Delta \zeta^2 = \Delta \tau^2 + \Delta \mathbf{r}_{\parallel}^2 + r^2 + r'^2$. Additionally, by making the change of variable $w = rr'/2s^2$ we get

$$W(x, x') = \frac{q}{2(rr')^{\frac{(D-2)}{2}} (2\pi)^{\frac{D+2}{2}}} \int_0^\infty dw w^{\frac{D-4}{2}} e^{-\frac{m^2 rr'}{2w} - \frac{(\Delta \zeta)^2 w}{2rr'}} \mathcal{I}(w, \kappa, q), \quad (\text{A.6})$$

where

$$\mathcal{I}(w, \kappa, q) = \int_{-\infty}^\infty \frac{q dh}{\kappa} e^{-\frac{q^2 h^2 rr'}{2w \kappa^2} + \frac{iqh}{\kappa} \Delta Z} \mathcal{S}(w, h, q), \quad \mathcal{S}(w, h, q) = \sum_{n=-\infty}^\infty e^{iq(n-h)\Delta \varphi} I_{q|n-h|}(w), \quad (\text{A.7})$$

with $h = \kappa \nu / q$.

The next step is to work out an expression for the function $\mathcal{I}(w, \kappa, q)$. Let us then start with the sum in n in Eq. (A.7), i.e.,

$$\mathcal{S}(w, h, q) = \sum_{n=-\infty}^\infty e^{iq(n-h)\Delta \varphi} I_{q|n-h|}(w). \quad (\text{A.8})$$

To develop the sum above we can make use of the very useful integral representation [66]

$$I_{\beta_n}(w) = \frac{1}{\pi} \int_0^\pi dy \cos(\beta_n y) e^{w \cos y} - \frac{\sin(\pi \beta_n)}{\pi} \int_0^\infty dy e^{-w \cosh y - \beta_n y}, \quad (\text{A.9})$$

where in our case $\beta_n = q|n-h|$. Thereby, substituting Eq. (A.9) into Eq. (A.8), and using the identity

$$\sum_{n=-\infty}^\infty e^{ibn} = 2\pi \sum_{n=-\infty}^\infty \delta(b - 2\pi n), \quad (\text{A.10})$$

the first term on the right-hand side of Eq. (A.9) becomes

$$\frac{1}{\pi} \sum_{n=-\infty}^\infty e^{iq(n-h)\Delta \varphi} \int_0^\pi dy \cos(\beta_n y) e^{w \cos y} = \frac{1}{q} \sum_l e^{-i2\pi l h} e^{w \cos(2\pi l/q - \Delta \varphi)}, \quad (\text{A.11})$$

with the discrete index l obeying the condition

$$-\frac{q}{2} + \frac{\Delta \varphi}{\varphi_0} \leq l \leq \frac{q}{2} + \frac{\Delta \varphi}{\varphi_0}, \quad (\text{A.12})$$

with $\varphi_0 = 2\pi/q$. If $\pm \frac{q}{2} + \frac{\Delta \varphi}{\varphi_0}$ are integers then the corresponding terms in the sum on the r.h.s of Eq. (A.11) should be taken with the coefficient 1/2. Hence, with the result in (A.11) we find for Eq. (A.8) that

$$\mathcal{S}(w, h, q) = \frac{1}{q} \sum_l e^{-i2\pi l h} e^{w \cos(2\pi l/q - \Delta \varphi)} - \frac{1}{\pi} \int_0^\infty dy e^{-w \cosh y} \sum_{n=-\infty}^\infty e^{iq(n-h)\Delta \varphi} \sin(\pi \beta_n) e^{-\beta_n y}. \quad (\text{A.13})$$

Upon using Eq. (A.13) to calculate the integral in Eq. (A.7) we get, for the first term on the right-hand side of Eq. (A.13), the expression

$$\begin{aligned} & \frac{1}{\kappa} \sum_l e^{w \cos(2\pi l/q - \Delta \varphi)} \int_{-\infty}^\infty dh e^{-\frac{q^2 h^2 rr'}{2w \kappa^2}} e^{-i2\pi l h + \frac{iqh}{\kappa} \Delta Z} \\ &= \frac{1}{\kappa} \sum_l e^{w \cos(2\pi l/q - \Delta \varphi)} e^{-\frac{w \Delta Z^2}{2rr'}} \int_{-\infty}^\infty dh e^{-\frac{rr' q^2}{2w \kappa^2} \left(h - i \frac{w \kappa \Delta Z_l}{q r r'}\right)^2} \\ &= \frac{1}{q} \sqrt{\frac{2w\pi}{rr'}} \sum_l e^{w \cos(2\pi l/q - \Delta \varphi)} e^{-\frac{w \Delta Z_l^2}{2rr'}}, \end{aligned} \quad (\text{A.14})$$

where $\Delta Z_l = \Delta Z - \bar{\kappa}l$ and $\bar{\kappa} = 2\pi\kappa/q$.

On the other hand, the second term on the right-hand side of Eq. (A.13) can be worked out as

$$\begin{aligned} & \int_{-\infty}^{\infty} \frac{qdh}{\kappa} e^{-\frac{q^2 h^2 r r'}{2w\kappa^2} + i\frac{qh\Delta Z}{\kappa}} \sum_{n=-\infty}^{\infty} e^{iq(n-h)\Delta\varphi} \sin(\pi\beta_n) e^{-\beta_n y} \\ &= \int_{-\infty}^{\infty} \frac{qdh}{\kappa} e^{-\frac{q^2 h^2 r r'}{2w\kappa^2} + i\frac{qh\Delta Z}{\kappa}} \int_{-\infty}^{\infty} dx e^{iqx\Delta\varphi} \sum_{n=-\infty}^{\infty} F(n) \sin(q\pi|x|) e^{-|x|y}, \end{aligned} \quad (\text{A.15})$$

where

$$F(n) = \delta[x - (n - h)], \quad (\text{A.16})$$

and we have made use of the delta function property $\int \delta(z - z_0)g(z) = g(z_0)$. By making use of Eq. (A.10) one can show that the sum in n of $F(n)$ above is given by

$$\sum_{n=-\infty}^{\infty} F(n) = \sum_{n=-\infty}^{\infty} e^{i2\pi n(x+h)}. \quad (\text{A.17})$$

Thus, the result in Eq. (A.17) allows us to re-write Eq. (A.15) as

$$\begin{aligned} & \sum_{n=-\infty}^{\infty} e^{-\frac{\Delta Z_n^2 w}{2rr'}} \int_{-\infty}^{\infty} \frac{qdh}{\kappa} e^{-\frac{q^2 r r'}{2w\kappa^2} \left(h - i\frac{\Delta Z_n \kappa w}{qrr'}\right)^2} \int_{-\infty}^{\infty} dx e^{ix\Delta\theta_n} \sin(q\pi|x|) e^{-q|x|y} \\ &= \sqrt{\frac{2w\pi}{rr'}} \sum_{n=-\infty}^{\infty} e^{-\frac{w\Delta Z_n^2}{2rr'}} \left\{ \frac{(q\pi - \Delta\theta_n)}{(q\pi - \Delta\theta_n)^2 + (qy)^2} + \frac{(q\pi + \Delta\theta_n)}{(q\pi + \Delta\theta_n)^2 + (qy)^2} \right\}, \\ &= \frac{1}{\pi} \sqrt{\frac{2w\pi}{rr'}} \sum_{n=-\infty}^{\infty} e^{-\frac{w\Delta Z_n^2}{2rr'}} M_{n,q}(\Delta\varphi, y), \end{aligned} \quad (\text{A.18})$$

where $\theta_n = 2\pi n + q\Delta\varphi$, $\Delta Z_n = \Delta Z + \bar{\kappa}n$ and

$$M_{n,q}(\Delta\varphi, y) = \frac{1}{2} \left\{ \frac{\left(\frac{q}{2} + \frac{\Delta\varphi}{\varphi_0} + n\right)}{\left(\frac{q}{2} + \frac{\Delta\varphi}{\varphi_0} + n\right)^2 + \left(\frac{y}{\varphi_0}\right)^2} - \frac{\left(-\frac{q}{2} + \frac{\Delta\varphi}{\varphi_0} + n\right)}{\left(-\frac{q}{2} + \frac{\Delta\varphi}{\varphi_0} + n\right)^2 + \left(\frac{y}{\varphi_0}\right)^2} \right\}. \quad (\text{A.19})$$

Therefore, the expression for the function $\mathcal{I}(w, \kappa, q)$ in (A.7) turns into

$$\begin{aligned} \mathcal{I}(w, \kappa, q) &= \sqrt{\frac{2w\pi}{rr'}} \left[\frac{1}{q} \sum_l e^{w\left(\cos(2\pi n/q - \Delta\varphi) - \frac{\Delta Z_l^2}{2rr'}\right)} \right. \\ &\quad \left. - \frac{1}{\pi^2} \sum_{n=-\infty}^{\infty} \int_0^{\infty} dy e^{-w\left(\cosh y + \frac{\Delta Z_n^2}{2rr'}\right)} M_{n,q}(\Delta\varphi, y) \right]. \end{aligned} \quad (\text{A.20})$$

So, with the expression (A.20), the Wightman function (A.6) is given by

$$\begin{aligned} W(x, x') &= \frac{m^{(D-1)}}{(2\pi)^{\frac{(D+1)}{2}}} \left[\sum_l f_{\frac{D-1}{2}}(m\sigma_l) \right. \\ &\quad \left. - \frac{q}{\pi^2} \sum_{n=-\infty}^{\infty} \int_0^{\infty} dy f_{\frac{D-1}{2}}(m\sigma_{y,n}) M_{n,q}(\Delta\varphi, y) \right], \end{aligned} \quad (\text{A.21})$$

where the function $f_\mu(x)$ is defined in terms of the Macdonald function, $K_\mu(x)$, i.e.,

$$f_\mu(x) = \frac{K_\mu(x)}{x^\mu}, \quad (\text{A.22})$$

and

$$\begin{aligned} \sigma_l^2 &= \left[\Delta\zeta^2 - 2rr' \cos(2\pi l/q - \Delta\varphi) + (\Delta Z - \bar{\kappa}l)^2 \right], \\ \sigma_{y,n}^2 &= \left[\Delta\zeta^2 + 2rr' \cosh(y) + (\Delta Z + \bar{\kappa}n)^2 \right]. \end{aligned} \quad (\text{A.23})$$

On the other hand, by making $m = 0$ in (A.6) and using (A.20), the Wightman function for the massless case is found to be

$$\begin{aligned} W(x, x') &= \frac{2^{\frac{(D-1)}{2}} \Gamma\left(\frac{D-1}{2}\right)}{2(2\pi)^{\frac{(D+1)}{2}}} \left[\sum_l \frac{1}{\sigma_l^{(D-1)}} \right. \\ &\quad \left. - \frac{q}{\pi^2} \sum_{n=-\infty}^{\infty} \int_0^\infty dy \frac{1}{\sigma_{y,n}^{(D-1)}} M_{n,q}(\Delta\varphi, y) \right]. \end{aligned} \quad (\text{A.24})$$

Note that this expression can also be obtained by taking the limit $m \rightarrow 0$ in Eq. (A.21).

B Summation formula for $M_{n,q}(\Delta\varphi, y)$

We are now interested in finding an expression for the summation in n of the function $M_{n,q}(\Delta\varphi, y)$. This can be done by using the psi function, $\psi(z)$, and its properties [66,67]. Thus, let us consider the following definition:

$$\psi(x + iy) = R(x, y) + iI(x, y), \quad (\text{B.1})$$

where

$$\begin{aligned} R(x, y) &= -\gamma + \sum_{n=0}^{\infty} \left[\frac{1}{n+1} - \frac{(n+x)}{(n+x)^2 + y^2} \right], \\ I(x, y) &= y \sum_{n=0}^{\infty} \frac{1}{(n+x)^2 + y^2}, \end{aligned} \quad (\text{B.2})$$

with $\gamma \simeq 0.58$ being the Euler number. Using Eq. (B.2) (see Ref. [67]) it is easy to verify that

$$\begin{aligned} R(1-x, y) - R(x, y) &= \sum_{n=-\infty}^{\infty} \frac{(n+x)}{(n+x)^2 + y^2} \\ &= \pi \cot(\pi x) \frac{\coth^2(\pi y) - 1}{\cot^2(\pi x) + \coth^2(\pi y)}. \end{aligned} \quad (\text{B.3})$$

Therefore, from Eq. (B.3), we get the final relation

$$\sum_{n=-\infty}^{\infty} \frac{(n+x)}{(n+x)^2 + y^2} = \pi \frac{\sin(2\pi x)}{\cosh(2\pi y) - \cos(2\pi x)}. \quad (\text{B.4})$$

The identity (B.4) is very important since it allows us to recover, from Eq. (A.20), the summation formula for the Bessel function $I_{qn}(w)$ in the limiting case $\kappa = 0$. This is the case where one considers the cosmic string spacetime only [64,65].

For $\kappa = 0$, the sum in n in Eq. (A.20) reduces to

$$\begin{aligned} M_q(\Delta\varphi, y) &= \sum_{n=-\infty}^{\infty} M_{n,q}(\Delta\varphi, y), \\ &= \frac{1}{2} \sum_{j=+,-} \sum_{n=-\infty}^{\infty} \frac{\left(\frac{q}{2} + \frac{j\Delta\varphi}{\varphi_0} + n\right)}{\left(\frac{q}{2} + \frac{j\Delta\varphi}{\varphi_0} + n\right)^2 + \left(\frac{y}{\varphi_0}\right)^2}. \end{aligned} \quad (\text{B.5})$$

In order to apply Eq. (B.4) for our case, let us make the change $x \rightarrow (q/2 + j\Delta\varphi/\varphi_0)$ and $y \rightarrow y/\varphi_0$. This provides

$$M_q(\Delta\varphi, y) = \frac{\pi}{2} \sum_{j=+,-} \frac{\sin(q\pi + jq\Delta\varphi)}{\cosh(qy) - \cos(q\pi + jq\Delta\varphi)}. \quad (\text{B.6})$$

In addition, by taking $\kappa = 0$ in Eq. (A.20) and using Eq. (B.6) we find

$$\begin{aligned} \mathcal{I}(w, q) &= \frac{\sqrt{2w\pi}}{r} e^{-\frac{w\Delta z^2}{2r^2}} \left[\frac{1}{q} \sum_n e^{w \cos(2\pi n/q - \Delta\varphi)} \right. \\ &\quad \left. - \frac{1}{2\pi} \sum_{j=+,-} \int_0^\infty dy \frac{\sin(q\pi + jq\Delta\varphi)}{\cosh(qy) - \cos(q\pi + jq\Delta\varphi)} e^{-w \cosh y} \right], \end{aligned} \quad (\text{B.7})$$

which is, up to the factor $(2w\pi/r^2)^{\frac{1}{2}} e^{-\frac{w\Delta z^2}{2r^2}}$ that comes from the integral in Eq. (A.7), the summation formula for Eq. (A.8) in the case $\kappa = 0$, when considering the Casimir energy density in the cosmic string spacetime [64, 65].

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